SADLER MATHEMATICS METHODS UNIT 3

WORKED SOLUTIONS

Chapter 6 The exponential function

Exercise 6A

Question 1

a $A = 1000e^{0.12t}$

When $t = 5$,

$A = 1000e^{0.6}$

$= \$1822.12$

b When $t = 10$,

$A = 1000e^{1.2}$

$= \$3320.12$

c When $t = 25$,

$A = 1000e^{3}$

$= \$20085.54$

Question 2

$\$27819.26 = P \times e^{0.08401}$

ClassPad solve,

$P = \$12500$

Question 3

$A = 100e^{-0.03t}$

When $t = 10$,

$A = 100e^{-0.3}$

$= 74.08$ left

$\therefore 25.92$ g has decayed (~ 26 g)

Question 4

a $S = 2000000e^{-0.15t}$

When $t = 0$,

$S = 2\,000\,000$

b When $t = 2$,

$S = 2000000e^{-0.3}$

$= 1481636$

$\approx 1\,500\,000$

c When $t = 4$,

$S = 2000000e^{-0.6}$

$= 1\,097\,623$

$\approx 1100\,000$

d When $t = 6$,

$S = 2000000e^{-0.9}$

$= 813139$

$\approx 800\,000$

Question 5

a $V = 75(1 - e^{-0.13t})$ m/s

When $t = 5$,

$V = 75(1 - e^{-0.13(5)})$

$= 35.8$ m/s

b When $t = 20$,

$V = 75(1 - e^{-0.13(20)})$

$= 69.4$ m/s

c When $t = 40$,

$V = 75(1 - e^{-0.13(40)})$

$= 74.6$ m/s

Question 6

a $$Y = 20 + \frac{40}{(e^{0.05x})}$$
$$60 = 20 + 40(e^{-0.05x})$$
$$x = 0$$

b $$Y = 20 + \frac{40}{(e^{0.05x})}$$
$$30 = 20 + 40(e^{-0.05x})$$
$$x = 27.73$$

c $$Y = 20 + \frac{40}{(e^{0.05x})}$$
$$21 = 20 + 40(e^{-0.05x})$$
$$x = 73.78$$

Question 7

$$N = \frac{3000}{1 + 2999e^{-0.4t}}$$
$$1000 = \frac{3000}{1 + 2999e^{-0.4t}}$$
$$t \approx 18$$

Question 8

a $$\frac{2000(e^{0.01\times 10\times 10} - 1)}{1 - e^{-0.01\times 10}}$$
$$= \$36\ 112.55$$

b $$154\ 000 = \frac{3000(e^{0.01\times 8\times t} - 1)}{1 - e^{-0.01\times 8}}$$
$$t = 19.98 \text{ (2 dp)}$$
$$\approx 20 \text{ years}$$

Exercise 6B

Question 1

e^x

Question 2

$7e^x$

Question 3

$3e^x$

Question 4

$6e^x$

Question 5

$9e^x$

Question 6

$-8e^x$

Question 7

$5e^{5x}$

Question 8

$7e^{7x}$

Question 9

$-2e^{-2x}$

Question 10

$15e^{3x}$

Question 11

$2e^{0.5x}$

Question 12

$e^{-0.5x}$

Question 13

$6e^x + 6x^2 + 6x$

Question 14

$$2e^x + \frac{1}{2}x^{-\frac{1}{2}}$$
$$= 2e^x + \frac{1}{2\sqrt{x}}$$

Question 15

$5e^{5x} + 2e^{2x}$

Question 16

$8e^{4x}$

Question 17

$6e^{3x} + 6e^{2x}$

Question 18

$15e^{3x} + 4x^3$

Question 19

$3e^{3x-1}$

Question 20

$2xe^{x^2+3}$

Question 21

$5e^{5x-1}$

Question 22

$(6x+2)(e^{3x^2+2x-1})$

Question 23

$$3x^2e^{x^3}$$

Question 24

$$\begin{aligned}&x\times 2e^{2x}+e^{2x}\times 1\\&=e^{2x}(2x+1)\end{aligned}$$

Question 25

$$\begin{aligned}f(x)&=x^3e^x\\f'(x)&=x^3e^x+e^x\times 3x^2\\&=e^x(x^3+3x^2)\\&=x^2e^x(3+x)\end{aligned}$$

Question 26

$$\begin{aligned}f(x)&=e^x\sqrt{x}\\f'(x)&=e^x\times\frac{1}{2}x^{-\frac{1}{2}}+\sqrt{x}\times e^x\\&=e^x\times\frac{1}{2\sqrt{x}}+\sqrt{x}e^x\\&=e^x\left(\frac{1}{2\sqrt{x}}+\frac{\sqrt{x}}{1}\right)\\&=\frac{e^x(1+2x)}{2\sqrt{x}}\end{aligned}$$

Question 27

$$\begin{aligned}f(x)&=\frac{e^x}{2x}\\f'(x)&=\frac{2x\times e^x-e^x\times 2}{4x^2}\\&=\frac{2e^x(x-1)}{4x^2}\\&=\frac{e^x(x-1)}{2x^2}\end{aligned}$$

Question 28

$$f(x) = e^x(1+2x)^3$$
$$f'(x) = e^x \times 3(1+2x)^2 \times 2 + (1+2x)^3 \times e^x$$
$$= (1+2x)^2 e^x (6+1+2x)$$
$$= e^x(1+2x)^2(2x+7)$$

Question 29

$$f(x) = e^x(1-2x)^5$$
$$f'(x) = e^x \times 5(1+2x)^4 \times (-2) + (1-2x)^5 e^x$$
$$= e^x(1-2x)^4(-10+1-2x)$$
$$= e^x(1-2x)^4(-9-2x)$$
$$= -e^x(1-2x)^4(2x+9)$$

Question 30

$$f(x) = e^{-3x}$$
$$f'(x) = -3e^{-3x}$$
$$= -\frac{3}{e^{3x}}$$

Question 31

$$y = e^{2x} + x^2$$
$$\frac{dy}{dx} = 2e^{2x} + 2x$$
When $x = 1$,
$$\frac{dy}{dx} = 2e^2 + 2$$
$$= 2(e^2 + 1)$$

Question 32

$$y = xe^x$$
$$\frac{dy}{dx} = x \times e^x + e^x \times 1$$
$$= e^x(x+1)$$
At $x = 1$,
$$\frac{dy}{dx} = 2e$$

Question 33

$y = 5e^{2x}$

$\frac{dy}{dx} = 10e^{2x}$

At $x = 0$,

$\frac{dy}{dx} = 10$

Equation of tangent

$y = 10x + c$

Using $(0, 5)$

$y = 10x + 5$

Question 34

Instantaneous rate of growth $\rightarrow$ derivative

$\frac{dA}{dt} = 0.08 \times 100e^{0.08t}$

$= 8e^{0.08t}$

a At $t = 1$,

$8e^{0.08} = \$8.67 \text{ / year}$

b At $t = 10$,

$8e^{0.8} = \$17.80 \text{ / year}$

c At $t = 20$,

$8e^{1.6} = \$39.62 \text{ / year}$

d At $t = 40$,

$8e^{3.2} = \$196.26 \text{ / year}$

Question 35

a $A_t = 100e^{-0.1t}$ tonnes

$A_0,\ t = 0$

$\therefore 100e^{0} = 100$

b When $t = 5$,

$100e^{-0.5} = 60.65$

$\therefore$ 61 tonnes

c $A_t = 100e^{-0.1t}$

$$\frac{dA}{dt} = 100 \times -(0.1)e^{-0.1t}$$
$$= -10e^{-0.1t}$$

When $t = 2$,

$$\frac{dA}{dt} = -10e^{-0.2}$$
$$= 8.187$$

$\therefore$ Falling at 8.19 tonnes/week.

d When $t = 5$,

$$\frac{dA}{dt} = -10e^{-0.5}$$
$$= -6.065$$

$\therefore$ Falling at 6.07 tonnes/week.

e When $t = 8$,

$$\frac{dA}{dt} = -10e^{-0.8}$$
$$= -4.49$$

$\therefore$ Falling at 4.49 tonnes/week.

Exercise 6C

Question 1

$$\frac{dA}{dt} = 2.5A$$

$$\begin{aligned} A &= A_0 e^{2.5t} \\ &= 50e^{2.5t} \end{aligned}$$

a When $t = 1$,

$$\begin{aligned} A &= 50e^{2.5} \\ &\approx 609 \end{aligned}$$

b When $t = 3$,

$$\begin{aligned} A &= 50e^{7.5} \\ &= 90402 \\ &\approx 90400 \end{aligned}$$

Question 2

$$\frac{dP}{dt} = 0.01P$$

$$P = 2000e^{0.01t}$$

a When $t = 10$,

$$\begin{aligned} P &= 2000e^{0.1} \\ &\approx 2210 \end{aligned}$$

b When $t = 50$,

$$\begin{aligned} P &= 2000e^{0.5} \\ &\approx 3297 \end{aligned}$$

Question 3

$Q = 150e^{0.03t}$

a When $t = 2$,

$$\begin{aligned} Q &= 150e^{0.03(2)} \\ &\approx 159 \end{aligned}$$

b When $t = 25$,

$$\begin{aligned} Q &= 150e^{0.03(25)} \\ &= 317.55 \\ &\approx 318 \end{aligned}$$

Question 4

$A = 20000e^{-0.1t}$

a When $t = 10$,

$$\begin{aligned} A &= 20000e^{-0.1(10)} \\ &\approx 7358 \end{aligned}$$

b When $t = 20$,

$$\begin{aligned} A &= 20000e^{-0.1(20)} \\ &\approx 2706.7 \\ &\approx 2707 \end{aligned}$$

Question 5

$$X = X_0 \times e^{0.5t}$$
$$6 \times 10^6 = X_0 e^{0.5(5)}$$
$$X_0 = \frac{6 \times 10^6}{e^{0.5(5)}}$$
$$= 492510$$
$$\Rightarrow X = 492510 \times e^{0.5t}$$

a When $t = 10$,

$$X = 492510 \times e^{0.5(10)}$$
$$= 73094965$$
$$\approx 7.3 \times 10^7$$

b When $t = 20$,

$$X = 492510 e^{10}$$
$$= 1.085 \times 10^{10}$$

Question 6

$$P = P_0 e^{0.025t}$$
$$2000 = P_0 e^{0.25}$$
$$P_0 = \frac{2000}{e^{0.25}}$$
$$= 1557.6$$

a When $t = 11$,

$$P = 1557.6 \times e^{0.275}$$
$$= 2050.628$$
$$\approx 2050$$

b When $t = 20$,

$$P = 1557.6 \times e^{0.5}$$
$$= 2568.05$$
$$\approx 2570$$

Question 7

$P_0 = 250$ million

$P = 250 \times e^{0.03t}$ million

a When $t = 10$,

$$\begin{aligned} P &= 250e^{0.3} \text{ million} \\ &= 337.46 \text{ million} \\ &\approx 340 \text{ million} \end{aligned}$$

b When $t = 50$,

$$\begin{aligned} P &= 250e^{1.5} \text{ million} \\ &= 1120.4 \text{ million} \\ &\approx 1120 \text{ million} \end{aligned}$$

Question 8

$P = 250 \times e^{0.025t}$ million

a When $t = 10$,

$$\begin{aligned} P &= 321 \text{ million} \\ &\approx 320 \text{ million} \end{aligned}$$

b When $t = 50$,

$$\begin{aligned} P &= 872.59 \text{ million} \\ &\approx 870 \text{ million} \end{aligned}$$

Question 9

$A_0 = 3,\ r = -0.12$

$A = 3e^{-0.12t}$

When $t = 20$,

$$\begin{aligned} A &= 3e^{-2.4} \\ &= 0.272 \text{ kg} \quad \text{or} \quad 272 \text{ g} \end{aligned}$$

Question 10

$P_0 = 5000,\ r = 0.11$
$P = 5000e^{0.11t}$
When $t = 25$,

$$\begin{aligned} P &= 5000e^{0.11\times 25} \\ &= \$78\,213.16 \end{aligned}$$

Question 11

$$\begin{aligned} 20000 &= A_0 e^{0.12\times 20} \\ &= \$1814.36 \end{aligned}$$

Question 12

$A_0 = 80,\ r = 0.05$
When $t = 100$,

$$\begin{aligned} A &= 80e^{0.05\times 100} \\ &= 11873 \\ &\therefore \$118.73 \end{aligned}$$

Question 13

$$\begin{aligned} A &= 80e^{0.08\times 100} \\ &= \$2384.77 \end{aligned}$$

Question 14

$P_0 = 10000,\ r = -0.05$
$P = 10000e^{-0.05t}$

a When $t = 5$,

$$\begin{aligned} P &= 10000e^{-0.05\times 5} \\ &= 7788 \\ &\approx 7800 \text{ frogs} \end{aligned}$$

b When $t = 10$,

$$\begin{aligned} P &= 10000e^{-0.05\times 10} \\ &= 6065 \\ &\approx 6100 \text{ frogs} \end{aligned}$$

Question 15

$r = 2\%,\ P = P_0 e^{0.02t}$

a 0.02

b $P_0 = 20$ million

$50 = 20e^{0.02t}$

$t = 45.81$ years

$\therefore 2000 + 46 = 2046$

Question 16

$P_0 = 1.5$ million, $k = 0.05$

$\therefore P = 1.5e^{0.05t}$ million

a

In 2025, $t = 25$

$P = 1.5e^{0.05\times 25}$

$= 5.2$

5.2 million

b

In 2050, $t = 50$

$P = 1.5e^{0.05\times 50}$

$= 18.3$

18.3 million

Question 17

$k = 1.2,\ P_0 \sim 1000$

$P = 1000e^{1.2t}$

a $10^6 = 10^3 e^{1.2t}$

$t = 5.76$

$\therefore \approx 5.8$

b $2 \times 10^6 = 10^3 e^{1.2t}$

$t = 6.33$

$\therefore \approx 6.3$

c $2000 = 10e^{1.2t}$

$4 = e^{1.2t}$

$t = 0.58$ hours

d $4000 = 10e^{1.2t}$

$4 = e^{1.2t}$

$t = 1.16$ hours

Question 18

$k = -0.25$

$P_0 = 2000$

$P = 2000e^{-0.25t}$

$P_4 = 2000e^{-1}$

$= 736$

$\therefore \sim 740$ rabbits

Question 19

$k = -0.24$

$S = S_0 e^{-0.24t}$

$0.45S_0 = S_0 e^{-0.24t}$

$0.45 = e^{-0.24t}$

$t = 3.3$ weeks

Exercise 6D

Question 1

$$\int 6e^{3x}dx$$
$$= 2\int 3e^{3x}dx$$
$$= 2e^{3x} + c$$

Question 2

$$\int 6e^{2x}dx$$
$$= 3\int 2e^{2x}dx$$
$$= 3e^{2x} + c$$

Question 3

$$\frac{1}{5}\int 5e^{5x}dx$$
$$= \frac{1}{5}e^{5x} + c$$

Question 4

$$\frac{1}{3}\int 9e^{9x}dx$$
$$= \frac{1}{3}e^{9x} + c$$
$$= \frac{1}{3}e^{9x} + c$$

Question 5

$$\frac{5}{3}\int 3e^{3x}dx$$
$$= \frac{5}{3}e^{3x} + c$$

Question 6

$$-5\int(-1)e^{-x}dx$$
$$=-5e^{-x}+c$$
$$=-\frac{5}{e^{x}}+c$$

Question 7

$$8\int\frac{1}{2}e^{\frac{x}{2}}dx$$
$$=8\sqrt{e^{x}}+c$$

Question 8

$$(-\frac{1}{2})\int(-2)e^{-2x}dx$$
$$=-\frac{1}{2}e^{-2x}+c$$
$$=-\frac{1}{2e^{2x}}+c$$

Question 9

$$\int(4e^{2x}+2x)dx$$
$$=2\int 2e^{2x}dx+\int 2xdx$$
$$=2e^{2x}+x^{2}+c$$

Question 10

$$\int(e^{3x}+e^{2x})dx$$
$$=\frac{1}{3}\int 3e^{3x}dx+\frac{1}{2}\int 2e^{x}dx$$
$$=\frac{1}{3}e^{3x}+\frac{1}{2}e^{2x}+c$$

Question 11

$$-\frac{3}{2}\int(-2)e^{-2x}dx$$
$$=-\frac{3}{2}e^{-2x}+c$$
$$=-\frac{3}{2e^{2x}}+c$$

Question 12

$$\int\left(4e^{-2x}+\frac{e^{2x}}{4}\right)dx$$
$$=(-2)\int(-2)e^{-2x}dx+\frac{1}{8}\int 2e^{2x}dx$$
$$=-2e^{-2x}+\frac{1}{8}e^{2x}+c$$
$$=-\frac{2}{e^{2x}}+\frac{e^{2x}}{8}+c$$

Question 13

$$\int 2xe^{x^2}dx$$
$$=e^{x^2}+c$$

Question 14

$$3\int 2e^{2x+1}dx$$
$$=3e^{2x+1}+c$$

Question 15

$$4\int(2xe^{x^2+5})dx$$
$$=4e^{x^2+5}+c$$

Question 16

$$\int_0^2 5e^x dx$$
$$= \left[5e^x\right]_0^2$$
$$= 5e^2 - 5$$
$$= 5(e^2 - 1)$$

Question 17

$$\frac{1}{5}\int_0^1 5e^{5x} dx$$
$$= \frac{1}{5}\left[e^{5x}\right]_0^1$$
$$= \frac{1}{5}(e^5 - e^0)$$
$$= \frac{(e^5 - 1)}{5}$$

Question 18

$$\int_1^2 (e^x + 2\times 2e^{2x})\ dx$$
$$= \left[e^x + 2e^{2x}\right]_1^2$$
$$= (e^2 + 2e^4) - (e^1 + 2e^2)$$
$$= e^2 + 2e^4 - e^1 - 2e^2$$
$$= 2e^4 - e^2 - e$$

Question 19

$$2\int_0^2 (x + \frac{1}{2}\times 2e^{2x})\ dx$$
$$= 2\left[\frac{x^2}{2} + \frac{e^{2x}}{2}\right]_0^2$$
$$= 2\left((2 + \frac{e^4}{2}) - (0 + \frac{e^0}{2})\right)$$
$$= 4 + e^4 - 1$$
$$= 3 + e^4$$

Question 20

$$\int_{-1}^{0} e^{-x} dx$$
$$= \left[-e^{-x}\right]_{-1}^{0}$$
$$= -e^{0} - (-e^{1})$$
$$= -1 + e$$
$$= e - 1$$

Question 21

$$6\int_{0}^{2} \left(2 \times \frac{1}{2} e^{\frac{1}{2}x} + x^{2} \right) dx$$
$$= 6\left[2e^{\frac{1}{2}x} + \frac{x^{3}}{3} \right]_{0}^{2}$$
$$= 6\left(\left(2e + \frac{8}{3} \right) - \left(2e^{0} + 0 \right) \right)$$
$$= 6\left(2e + \frac{8}{3} - 2 \right)$$
$$= 12e + 4$$

Question 22

a $\dfrac{dA}{dt} = 5e^{2t},\ A = 3,\ t = 0$

$$A(t) = \frac{5}{2}\int 2e^{2t}\,dt$$
$$= \frac{5e^{2t}}{2} + c$$

$$3 = \frac{5e^{0}}{2} + c$$
$$c = \frac{1}{2}$$

$$A(t) = \frac{5}{2}e^{2t} + \frac{1}{2}$$
$$= \frac{1+5e^{2t}}{2}$$

b $A = \dfrac{1+5e}{2}$

Question 23

a $f'(x) = 6(x^2 - 2e^{3x})$

$$f(x) = 6\left(\frac{1}{3}x^3 - \frac{2}{3}e^{3x}\right) + c$$
$$= 2x^3 - 4e^{3x} + c$$

$$f(0) = 0 - 4 + c = 1$$
$$c = 5$$
$$f(x) = 2x^3 - 4e^{3x} + 5$$

b

$$f(2) = 16 - 4e^6 + 5$$
$$= 21 - 4e^6$$

Question 24

a $$\int_0^3 e^x dx = \left[e^x\right]_0^3$$
$$= e^3 - e^0$$
$$= e^3 - 1$$
$$= 19.1 \text{ units}^2$$

b $$\int_0^3 e^x - e \ dx = \left[e^x - ex\right]_0^3$$
$$= (e^3 - 3e) - (e^0 - 0)$$
$$= e^3 - 3e - 1 \text{ units}^2$$

Question 1

$y = (2x-1)(3x+2)$

$\frac{dy}{dx} = (2x-1)(3) + (3x+2)(2)$

$= 6x - 3 + 6x + 4$

$= 12x + 1$

When $x = 1$,

$\frac{dy}{dx} = 12(1) + 1$

$= 13$

$\therefore$ Tangent is of the form $y = 13x + c$

Using (1, 5)

$5 = 13(1) + c$

$c = -8$

$\therefore$ Equation of tangent is $y = 13x - 8$

Question 2

a $\frac{dy}{dx} = 5(x+2)^4 \times 1$

$= 5(x+2)^4$

b $\frac{dy}{dx} = 5(2x+1)^4 \times 2$

$= 10(2x+1)^4$

c $\frac{dy}{dx} = \frac{(x+5)\times 1 - (x-5)\times 1}{(x+5)^2}$

$= \frac{x+5-x+5}{(x+5)^2}$

$= \frac{10}{(x+5)^2}$

d $\frac{dy}{dx} = \frac{(x+5)\times 5 - (5x-1)\times 1}{(x+5)^2}$

$= \frac{5x+25-5x+1}{(x+5)^2}$

$= \frac{26}{(x+5)^2}$

e $\frac{dy}{dx} = 12x^2 - e^x$

f $\frac{dy}{dx} = 5e^{5x} + 5$

Question 3

$\frac{dy}{dx} = 3ax^2$

When $x = 5$,

$\frac{dy}{dx} = 3a \times 5^2 = 30$

$75a = 30$

$a = 0.4$

$y = 0.4x^3$

$b = 0.4(5)^3$

$= 50$

Question 4

a $v = e^{0.1t}$

$a = \frac{dv}{dt} = 0.1e^{0.1t}$

When $t = 0$,

$a = 0.1e^{0.1(0)}$

$\quad = 0.1 \text{ m/s}^2$

b When $t = 20$,

$a = 0.1e^{0.1(20)}$

$\quad = 0.1e^2$

$\quad = 0.739 \text{ m/s}^2$

c $x = \int v dt$

$\quad = \int e^{0.1t} dt$

$\quad = 10\int 0.1e^{0.1t} dt$

$\quad = 10e^{0.1t} + c$

When $t = 0$,

$x = 10e^{0.1(0)} + c$

$12 = 10 + c$

$c = 2$

$\therefore c = 10e^{0.1t} + 2$

When $t = 10$,

$x = 10e^{0.1(10)} + 2$

$\quad = 10e + 2$

$\quad = 29.183 \text{ m}$

Question 5

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\left[(x+h)^2+3(x+h)\right]-\left[x^2+3x\right]}{h}$$

$$=\lim_{h\to 0}\left(\frac{x^2+2hx+h^2+3x+3h-x^2-3x}{h}\right)$$

$$=\lim_{h\to 0}\left(\frac{2hx+h^2+3h}{h}\right)$$

$$=\lim_{h\to 0}\frac{h(2x+h+3)}{h}$$

$$=2x+3$$

Question 6

$$y=x^3+3x^2-10x=0$$

$$x(x^2+3x-10)=0$$

$$x(x+5)(x-2)=0$$

$$x=0,\ 2,-5$$

$\therefore$ (0, 0), (2, 0) and (–5, 0)

When $x=-5$,

$$\frac{dy}{dx}=3(-5)^2+6(-5)-10$$

$$=35$$

$\therefore$ The gradient at $(-5,\ 0)$ is 35.

$$\frac{dy}{dx}=3x^2+6x-10$$

When $x=0$,

$$\frac{dy}{dx}=3(0)^2+6(0)-10$$

$$=-10$$

$\therefore$ The gradient at (0, 0) is -10.

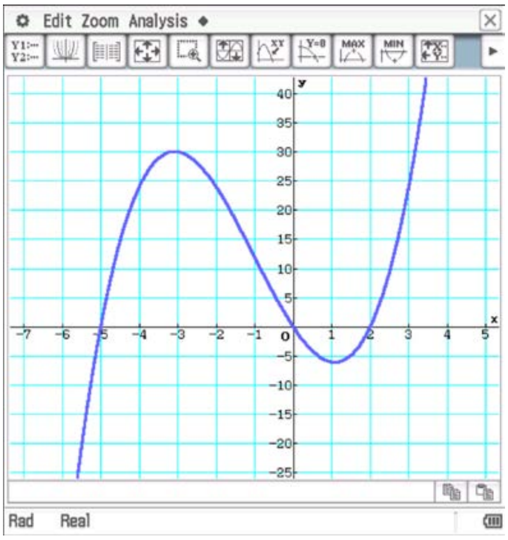
When $x=2$,

$$\frac{dy}{dx}=3(2)^2+6(2)-10$$

$$=14$$

$\therefore$ The gradient at (2, 0) is 14.

$$\int_{-5}^{0}(x \ +3x^2-10x)dx-\int_{0}^{2}(x^3+3x^2-10x)dx$$
$$=\left[\frac{x^4}{4}+x^3-5x^2\right]_{-5}^{0}-\left[\frac{x^4}{4}+x^3-5x^2\right]_{0}^{2}$$
$$=\left(0-(5^4+(-5)^3-5(-5)^2\right)-\left(\frac{2^4}{4}+2^3-5\times 2^2-0\right)$$
$$=93.75-(-8)$$
$$=101.75$$
$\therefore$ Total area enclosed 101.75 units^2.



Question 7

$$2y=-x+8 \qquad \rightarrow \qquad m_1=-\frac{1}{2}$$
$$\frac{dy}{dx}=2ax$$
When $x=-1$,
$$\frac{dy}{dx}=2a(-1)=2$$
$$-2a=2$$
$$a=-1$$
$$y=-x^2+5$$
$$b=-(-1)^2+5$$
$$=4$$

Question 8

a $\int_2^{10} x\,dx$

$= \left[\frac{x^2}{2}\right]_2^{10}$

$= \frac{100}{2} - \frac{4}{2}$

$= 48$

b $\int_1^2 \frac{1}{x^2}\,dx$

$= \left[-\frac{1}{x}\right]_1^2$

$= \left(-\frac{1}{2} - \left(-\frac{1}{1}\right)\right)$

$= \frac{1}{2}$

c $\int_0^1 e^x dx$

$= \left[e^x\right]_0^1$

$= e^1 - e^0$

$= e - 1$

d $\int_0^1 6e^{2x} dx$

$= 3\int_0^1 2e^{2x} dx$

$= 3\left[e^{2x}\right]_0^1$

$= 3(e^2 - e^0)$

$= 3(e^2 - 1)$

$= 3e^2 - 3$

e $\int_{-1}^2 (3x^2 + 4x)dx$

$= \left[x^3 + 2x^2\right]_{-1}^2$

$= (2^3 + 2 \times 2^2) - ((-1)^3 + 2(-1)^2)$

$= 16 - (1)$

$= 15$

f $\int_2^3 \frac{4x}{(x^2-3)^2}\,dx$

$= \int_2^3 4x(x^2-3)^{-2}\,dx$

$= 2\int_2^3 2x(x^2-3)^{-2}dx$

$= 2\left[-(x^2-3)^{-1}\right]_2^3$

$= 2\left(-(3^2-3)^{-1}-(-(2^2-3)^{-1}\right)$

$= 2(-\frac{1}{6}-(-\frac{1}{1}))$

$= 2\times\frac{5}{6}$

$= \frac{5}{3}$

Question 9

a $\frac{dy}{dx} = 6x^2 + \frac{1}{2}\times 4x^{-\frac{1}{2}}$

$= 6x^2 + \frac{2}{\sqrt{x}}$

b $\frac{dy}{dx} = 3x^2 + e^x$

c $\frac{dy}{dx} = \frac{(x+3)\times 2-(2x-1)\times 1}{(x+3)^2}$

$= \frac{2x+6-2x+1}{(x+3)^2}$

$= \frac{7}{(x+3)^2}$

d $\frac{dy}{dx} = x^4\times e^x + e^x\times 4x^3$

$= x^3e^x(x+4)$

e $\frac{dy}{dx} = 5(2x^3+4\sqrt{x})^4\times(6x^2+\frac{1}{2}\times 4x^{-\frac{1}{2}})$

$= 5(2x^3+4\sqrt{x})^4\left(6x^2+\frac{2}{\sqrt{x}}\right)$

f $\dfrac{d}{dx}\int_5^x \dfrac{e^{5t}}{t}\,dt = \dfrac{e^{5x}}{x}$

Question 10

a $\dfrac{1}{2}(1+2)\times 5 = 7.5\text{ km}$

b $7\dfrac{1}{2}+10\times 2 = 27.5\text{ km}$

c $27\dfrac{1}{2}+\dfrac{1}{2}\times 4\times 2 = 31.5\text{ km}$

Question 11

$$\begin{aligned} v &= \frac{200}{3}(1\times e^{-0.15(5)}) \\ &= 35.2\text{ m/s} \end{aligned}$$

$$\lim_{t\to\infty}\left(1-\frac{1}{e^{0.15t}}\right)=1$$

$$\frac{200}{3}\times 1 = \frac{200}{3}\text{ m/s}$$

Question 12

$$\frac{dy}{dx} \approx \frac{\delta y}{\delta x}$$

$$\begin{aligned} S &= 2\pi r^2 + 2\pi r \times 20 \\ &= 2\pi r^2 + 40\pi r \end{aligned}$$

$$\frac{dS}{dr} = 4\pi r + 40\pi$$

$$\begin{aligned} \delta S &\approx (4\pi r + 40\pi)\delta r \\ &\approx (4\times\pi\times 10 + 40\pi)0.2 \\ &\approx 80\pi\times 0.2 \\ &\approx 16\pi\text{ cm}^2 \end{aligned}$$

Question 13

$$\frac{dP}{dt} = 0.08P,\ P_0 = 500$$
$$\Rightarrow P = 500e^{0.08t}$$

a $P = 500e^{0.08(5)}$
$= 745.91$
$\therefore \$745.91$

b $P = 500e^{0.08(15)}$
$= 1660.06$
$\therefore \$1660.06$

Question 14

a $\frac{dT}{dt} = -28.5e^{-0.3t}$

When $t = 1$,

$$\frac{dT}{dt} = -28.5e^{-0.3}$$
$$= -21.1$$

$\therefore T$ is falling by 21.1°C/min.

b When $t = 3$,

$$\frac{dT}{dt} = -28.5e^{-0.3\times 3}$$
$$= -11.6$$

$\therefore T$ is falling by 11.6°C/min.

c When $t = 15$,

$$\frac{dT}{dt} = -28.5e^{-0.3\times 15}$$
$$= -0.3$$

$\therefore T$ is falling by 0.3°C/min.

Question 15

a $y = x^2 e^x$

x and y intercepts:

x-intercepts, $y = 0$

$$x^2 e^x = 0$$

$$x^2 = 0 \quad \text{or} \quad e^x = 0$$

$$x = 0 \qquad\qquad \text{no such } x$$

$(0,0)$ is the x-intercept

y-intercept, $x = 0$

$$\begin{aligned} y &= x^2.e^x \\ &= 0^2.e^0 \\ &= 0 \end{aligned}$$

$(0,0)$ is also the y-intercept

Co-ordinates of turning points:

$$\frac{dy}{dx} = x^2 e^x + e^x.2x$$

$$0 = xe^x(x+2)$$

$$xe^x = 0 \quad \text{or} \quad x+2=0$$

$$x = 0 \quad \text{or} \quad \mathrm{e}^x = 0 \quad \text{or} \quad x = -2$$

$$\text{no such } x$$

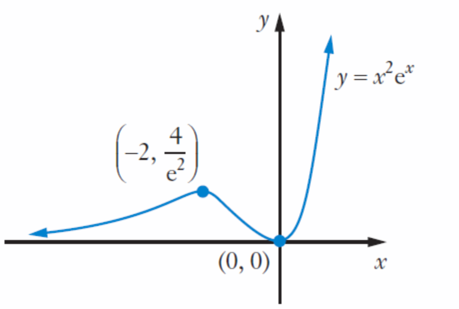
When $x = 0, y = 0$

$\Rightarrow (0,0)$ is a turning point

When $x = -2$,

$$\begin{aligned} y &= (-2)^2 e^{-2} \\ &= \frac{4}{e^2} \end{aligned}$$

$\Rightarrow (-2, \frac{4}{e^2})$ is a turning point



As $x \to \infty, y \to \infty \ \ (\infty^2.e^\infty)$

As $x \to -\infty, y \to 0 \ \ ((-\infty)^2.\frac{1}{e^\infty})$

b $y=\dfrac{e^x}{x^2}$

y-intercept :

$y=\dfrac{e^0}{0^2}\Rightarrow$ no y-intercept exists

The graph is asymptotic at $x=0$

x-intercept

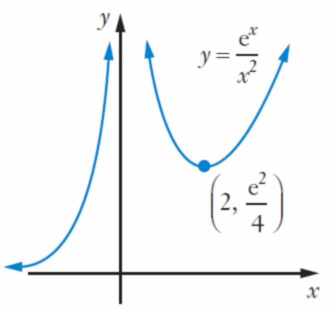
$0=\dfrac{e^x}{x^2}\Rightarrow$ No x-intercept exists as $e^x\neq 0$

Stationary points

$\dfrac{dy}{dx}=\dfrac{x^2e^x-e^x2x}{x^4}$

$0=\dfrac{xe^x(x-2)}{x^4}$

$x=0\quad$ or $\quad x=2\qquad (e^x\neq 0)$



At $x=2$,

$y=\dfrac{e^2}{2^2}$

$=\dfrac{e^2}{4}$

As $x\to\infty, y\to\infty$

As $x\to-\infty, y\to 0$

c y-intercept

$$y=\frac{1}{1+e^0}$$
$$=\frac{1}{2}$$
$$(0,\frac{1}{2})$$

x-intercept

$$0=\frac{1}{1+e^x}$$

No such x $\therefore$ no x-intercepts

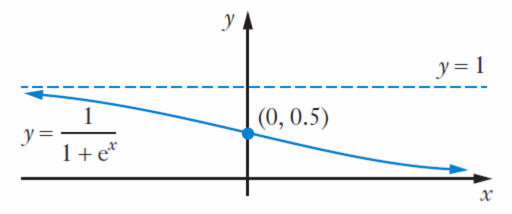
Stationary points

$$\frac{dy}{dx}=-1(1+e^x)^{-2}.e^x$$
$$0=\frac{-e^x}{(1+e^x)^2}$$

$-e^x\neq 0 \Rightarrow$ no stationary points

As $x\to\infty, y\to 0$

As $x\to-\infty, y\to 1$



Question 16

Point of intersection

By ClassPad, $y = 6 + \sqrt{x}$ and $4y + x = 56$ intersect at (16, 10).

$$\therefore \int_0^{16} (6 + \sqrt{x})dx$$

$$= \left[6x + \frac{2}{3}x^{\frac{3}{2}} \right]_0^{16}$$

$$= 138\frac{2}{3}$$

$$4y + x = 56$$

$$y = \frac{56 - x}{4}$$

$$= 14 - \frac{x}{4}$$

$$\int_{16}^{40} \left(14 - \frac{x}{4} \right) dx$$

$$= \left[14x - \frac{x^2}{8} \right]_{16}^{40}$$

$$= 168$$

$$\text{Area } = \left(168 + 138\frac{2}{3} \right) \times 2$$

$$= 613\frac{1}{3}$$

$\therefore 613 \text{ cm}^2$ (nearest cm)